Unified Panel Flutter Theory with Viscous Damping Effects

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The idea of a unified panel flutter theory, investigated previously by the author, is amplified to make the inclusion of damping effects possible. The paper attempts to study the combined effects of the aerodynamic and structural types of damping on the flutter boundaries for simply supported panels in high supersonic Mach number flows. Classical plate theory and two-dimensional first-order aerodynamics are used. A closed-form rather than modal approach is used in the solution of the partial differential equation in the analysis to avoid convergence problems. The standing and traveling wave theories of panel flutter are compared. The dual nature of damping (stabilizing and destabilizing) is exposed. While the trends are similar to those already established for isotropic panels, the variables compared are generically defined to account for both the isotropic as well as the orthotropic properties of the panels.

Nomenclature

A	$=(k_0-2n^2D^*)(a_0/b_0)^2$
$(a,b),(a_0,b_0)$	= dimensions of panel in Cartesian and affine space, respectively
c,c_r	= wave speed and reference wave speed, respectively
D^*	= generalized rigidity ratio
D_{ij}	= elastic parameters
G_s	= structural damping
g_a,g_i,g_s,g_T	= generic aerodynamic, actual structural damping coefficient of ith mode, structural, and total damping coefficients, respectively
h	= panel thickness
i	$=\sqrt{-1}$
j,m,n	= integers
K	= spring foundation stiffness
k	= generic eigenvalue
k_0,k_{0_y}	= generic buckling coefficients
k_s	= spring foundation, parameter
k_1,k_2	= real and imaginary parts of k, respectively
ℓ_0	= wavelength
M	= Mach number
$(N_x,N_y),(N_{x_0},N_{y_0})$	= midplane stresses (+ in compression) in Cartesian and affine space, respectively
t	= time
U	= airflow velocity
w,W	= panel deflections
$(x,y,z),(x_0,y_0,z_0)$	= Cartesian and affine coordinates, respectively
α	= generic decay rate of system
λ	= generic dynamic pressure parameter
$\xi,\xi_0,\eta,\tau,\tau_0$	= nondimensionalized variables in affine space
ρ,ρ_∞	= panel mass density and air mass density, respectively
Ω	= generic response of system
ω_{cr},λ_{cr}	= flutter ω and λ, respectively
ω_i	= frequency of ith mode
ω_r	= reference frequency

Superscript

~	= generic quantities for low aspect ratio panels

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Introduction

PERHAPS the most controversial topic in the general field of vibration is damping. The variety of damping expressions representing the same physical system in the literature seems to indicate a general lack of agreement about a satisfactory mathematical model. Consequently, generally accepted conclusions about the damping effects on even the simplest vibrating system are hard to come by.

In flutter, particularly panel flutter, the situation is further complicated by the existence of aerodynamic damping and several elastic and aerodynamic parameters. It would appear then that the main reasons for restricting the damping analyses in panel flutter largely to membranes and isotropic panels are as follows. First is the fact that the partial differential equation representing the isotropic panel or membrane contains at most one elastic parameter and, hence, is easier to handle than its anisotropic counterpart, which contains at least four elastic parameters. Second is that, in view of the existence of so many parameters (elastic and aerodynamic) in the anisotropic analysis, understanding the proper trends would call for the infusion of a new idea in the solution process. Affine transformation of the physical space not only supplies this new idea,[1-3] but makes a unified panel flutter theory of orthotropic and isotropic panels a reality.[4] The transformation, which is similar to the Prandtl-Glauert transformation[5] used in compressible aerodynamics, was also used by Brunelle and Oyibo[6] in the static instability analysis and the Virtual Work Theorem formulation.

Several authors have treated damping in panel flutter (see, e.g., Refs. 7-16). Johns and Parks[7] used the hysteretic structural damping representation in their analysis to show the destabilizing effects of damping. Ellen[8] tried to classify structural damping and showed which classes are always stabilizing using spatial derivative arguments. Houbolt,[9] the first to introduce the idea of aerodynamic damping, showed the stabilizing effects of aerodynamic damping. Dowell[10] showed the effects of damping on low aspect ratio panel flutter. Dugundji,[11] in his elaborate analysis, showed the combined effects of structural and aerodynamic damping on panel flutter. Reference 11 used the viscous representation for structural damping, which has an analytic advantage of grouping the structural and aerodynamic damping together as "total" damping (the last part of the paper treated various types of damping[23]). References 12 and 13 did include damping in their treatment of the orthotropic panel flutter, but the presence of so many parameters seems to interfere with the acquisition of insight that should be gained from the analyses (see Refs. 17-20, 22, and 24 for other panel flutter analyses).

The present analysis is basically a continuation of what was started in Ref. 4. By defining generic structural and aerodynamic variables via the affine transformations, this paper attempts to study the damping effects on the flutter boundaries for isotropic and orthotropic panels subjected to supersonic Mach number flows on one surface in a single analysis. Viscous structural and aerodynamic damping representations are used. Although classical plate theory and two-dimensional first-order aerodynamics are used, a closed-form, rather than a modal, approach is used in the solution of the partial differential equation to avoid convergence problems. The stabilizing and destabilizing effects of the chosen damping representations are exposed. Traveling and standing wave theories are also compared.

Statement of Problem

The analysis considers a flat, rectangular orthotropic panel of length a, width b, and a uniform thickness h, shown in Fig. 1. The panel is hinged at the edges and subjected to a supersonic airflow U over its top surface and midplane force intensities N_x and N_y. In addition, it has a viscous structural damping G_s (N-s m^{-3}) and rests on a spring foundation K(N-m^{-3}).

Equations of Motion

Using classical plate and first-order two-dimensional aerodynamic theories, the partial differential equation and the boundary conditions for the system in the physical space are

$$D_{11}\frac{\partial^4 w}{\partial x^4}+2(D_{12}+2D_{66})\frac{\partial^4 w}{\partial x^2\partial y^2}+D_{22}\frac{\partial^4 w}{\partial y^4}$$
$$+N_x\frac{\partial^2 w}{\partial x^2}+N_y\frac{\partial^2 w}{\partial y^2}+\rho h\frac{\partial^2 w}{\partial t^2}+G_s\frac{\partial w}{\partial t}$$
$$+\frac{\rho_\infty U^2}{(M^2-1)^{1/2}}\left[\frac{\partial w}{\partial x}+\frac{1}{U}\frac{(M^2-2)}{(M^2-1)}\frac{\partial w}{\partial t}\right]+Kw=0 \tag{1}$$
$$w(a,y,t)=\frac{\partial^2 w}{\partial x^2}(0,y,t)=\frac{\partial^2 w}{\partial x^2}(a,y,t)$$
$$=\frac{\partial^2 w}{\partial y^2}(x,0,t)=\frac{\partial^2 w}{\partial y^2}(x,b,t)=0$$
$$w(0,y,t)=w(x,0,t)=w(x,b,t)=0$$

where the aerodynamics used here are reasonably accurate for high supersonic Mach numbers ($M>1.6$).

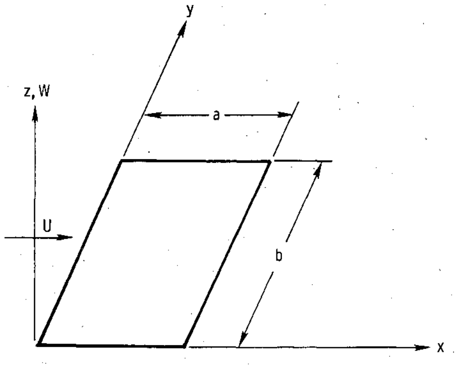

Fig. 1. A rectangular orthotropic panel having viscous structural damping and resting on a spring foundation.

Affine Transformations and Generic Variables

Consider the transformations,

$$x=(D_{11})^{1/4}x_0, \qquad y=(D_{22})^{1/4}y_0 \tag{2}$$

When Eqs. (2) are substituted into Eqs. (1), the following are obtained,†

$$\frac{\partial^4 w}{\partial x_0^4}+2D^*\frac{\partial^4 w}{\partial x_0^2\partial y_0^2}+\frac{\partial^4 w}{\partial y_0^4}+N_{x_0}\frac{\partial^2 w}{\partial x_0^2}+N_{y_0}\frac{\partial^2 w}{\partial y_0^2}$$
$$+\frac{\rho_\infty U^2}{[D_{11}(M^2-1)^2]^{1/4}}\frac{\partial w}{\partial x_0}+\rho h\frac{\partial^2 w}{\partial t^2}$$
$$+\left[G_s+\frac{\rho_\infty U(M^2-2)}{(M^2-1)^{3/2}}\right]\frac{\partial w}{\partial t}+Kw=0 \tag{3}$$
$$w(a_0,y_0,t)=\frac{\partial^2 w}{\partial x_0^2}(0,y_0,t)=\frac{\partial^2 w}{\partial x_0^2}(a_0,y_0,t)$$
$$=\frac{\partial^2 w}{\partial y_0^2}(x_0,0,t)=\frac{\partial^2 w}{\partial y_0^2}(x_0,b_0,t)=0$$
$$w(0,y_0,t)=w(x_0,0,t)=w(x_0,b_0,t)=0$$

where

$$D^*\triangleq(D_{12}+2D_{66})/(D_{11}D_{22})^{1/2},$$
$$N_{x_0}\triangleq N_x/(D_{11})^{1/2}, \qquad N_{y_0}\triangleq N_y/(D_{22})^{1/2} \tag{4}$$

Further consider the transformations,

$$x_0=a_0\xi, \quad y_0=b_0\eta, \quad \omega_r\triangleq[\pi^4/\rho h a_0^4]^{1/2}, \quad \tau_0=\omega_r t \tag{5}$$

Substituting Eqs. (5) into Eqs. (3) results in the following

$$\frac{\partial^4 w}{\partial\xi^4}+2D^*\left(\frac{a_0}{b_0}\right)^2\frac{\partial^4 w}{\partial\xi^2\partial\eta^2}+\left(\frac{a_0}{b_0}\right)^4\frac{\partial^4 w}{\partial\eta^4}$$
$$+k_0\pi^2\left(\frac{a_0}{b_0}\right)^2\frac{\partial^2 w}{\partial\xi^2}+k_{0_y}\pi^2\left(\frac{a_0}{b_0}\right)^2\frac{\partial^2 w}{\partial\eta^2}+\lambda\frac{\partial w}{\partial\xi}$$
$$+\pi^4\frac{\partial^2 w}{\partial\tau_0^2}+\pi^4 g_T\frac{\partial w}{\partial\tau_0}+\pi^4 k_s w=0 \tag{6}$$
$$w(1,\eta,\tau_0)=\frac{\partial^2 w}{\partial\xi^2}(0,\eta,\tau_0)=\frac{\partial^2 w}{\partial\xi^2}(1,\eta,\tau_0)$$
$$=\frac{\partial^2 w}{\partial\eta^2}(\xi,0,\tau_0)=\frac{\partial^2 w}{\partial\eta^2}(\xi,1,\tau_0)=0$$
$$w(0,\eta,\tau_0)=w(\xi,0,\tau_0)=w(\xi,1,\tau_0)=0$$

where

$$k_0\triangleq\frac{N_{x_0}b_0^2}{\pi^2}, \quad k_{0_y}\triangleq\frac{N_{y_0}a_0^2}{\pi^2}, \quad \lambda\triangleq\frac{\rho_\infty U^2 a_0^3}{[D_{11}(M^2-1)^2]^{1/4}} \tag{7}$$
$$g_T=\frac{1}{\pi^2}\left[G_s+\frac{\rho_\infty U(M^2-2)}{(M^2-1)^{3/2}}\right]\frac{a_0^4}{[\rho h a_0^4]^{1/2}}, \quad k_s\triangleq\frac{Ka_0^4}{\pi^4}$$

Equations (6), therefore, are the partial differential equation and the boundary conditions for the panel in the affine space, subject to the generic variables in Eqs. (4) and (7). The generic

†Similar transformations, $x=x_0$, $y=(D_{22}/D_{11})^{1/4}y_0$, or $x=(D_{11}/D_{22})^{1/4}x_0$, $y=y_0$ produce similar results. Although Eqs. (2) do not preserve lengths $[a=(D_{11})^{1/4}a_0, b=(D_{22})^{1/4}b_0]$, they are chosen because of their symmetry.

variable D^*, called the generalized rigidity ratio, is predicted to have a closed interval, $0 \le D^* \le 1$, for all orthotropic panels (for isotropic panels, $D^* = 1$) (see Refs. 1-4, 21); and k_0 and k_{0_y} are the buckling coefficients in the x_0 and y_0 directions, respectively.

Damping Representations

The damping used in the analysis consists of a viscous structural and an aerodynamic representation. Although the viscous structural damping has a general form, $G_s(\partial^{n+1} w/\partial t \partial x_i^n)$, where x_i is a spatial coordinate in the surface, the $n=0$ is chosen for the analytic advantage of being able to group the structural and aerodynamic damping terms together as the "total" damping term. Thus the aerodynamic damping coefficient g_a first introduced by Houbolt[9] is combined with the effective viscous structural damping coefficient to obtain the total damping coefficient, g_T. The damping coefficients in the affine space are defined as:

$$g_a \triangleq \frac{\rho_\infty U(M^2-2)\omega_r a_0^4}{\pi^4 (M^2-1)^{3/2}}, \quad g_s \triangleq \frac{G_s \omega_r a_0^4}{\pi^4} \tag{8}$$

where $G_s = g_j \omega_j \rho h$ for the jth mode (G_s is an assumed constant viscous structural damping) and $g_j = 2\zeta_j$, ζ_i being the critical damping ratio of any mode with the frequency ω_j. The reference frequency ω_r, represents the lowest natural frequency for the panel in the affine space ($\lambda = N_{x_0} = N_{y_0} = g_T = K = 0$). From Eqs. (7) and (8),

$$g_T = g_a + g_s \tag{9}$$

Solution of the Partial Differential Equation

In this analysis the closed-form solutions are sought for Eq. (6) in order to avoid the convergence problems of approximate methods. Therefore consider the mode function given by

$$w(\xi,\eta,\tau_0) = W(\xi)\sin(n\pi\eta)e^{\Omega\tau_0} \tag{10}$$

When Eq. (10) is substituted into Eq. (6), the following are obtained:

$$\frac{\partial^4 W}{\partial \xi^4} + A\pi^2 \frac{\partial^2 W}{\partial \xi^2} + \lambda \frac{\partial W}{\partial \xi} + kW = 0$$
$$W(0) = (1) = \frac{\partial^2 W}{\partial \xi^2}(0) = \frac{\partial^2 W}{\partial \xi^2}(1) = 0 \tag{11}$$

where

$$A \triangleq (k_0 - 2D^* n^2)(a_0/b_0)^2 \tag{12a}$$

$$k \triangleq \pi^4 [\Omega^2 + g_T \Omega + n^4 (a_0/b_0)^4 - k_{0_y} n^2 (a_0/b_0)^2 + k_s] \tag{12b}$$

and

$$k = k_1 + ik_2, \qquad \Omega = \alpha + i\left(\frac{\omega}{\omega_r}\right) \tag{13}$$

where k_1, k_2, α, ω, ω_r are real variables and $i = \sqrt{-1}$.

The solution to Eqs. (11) has the following general form,

$$W(\xi) = a_1 e^{p_1 \xi} + a_2 e^{p_2 \xi} + a_3 e^{p_3 \xi} + a_4 e^{p_4 \xi} \tag{14}$$

When Eq. (14) is substituted into Eqs. (11), the following frequency equation is obtained for a nontrivial solution

$$8\sigma^2 \beta\delta(\cosh 2\sigma - \cosh\beta\cos\delta) + \sin\delta\sinh\beta[(\beta^2+\delta^2)^2 + 4\sigma^2(\delta^2-\beta^2)] = 0 \tag{15}$$

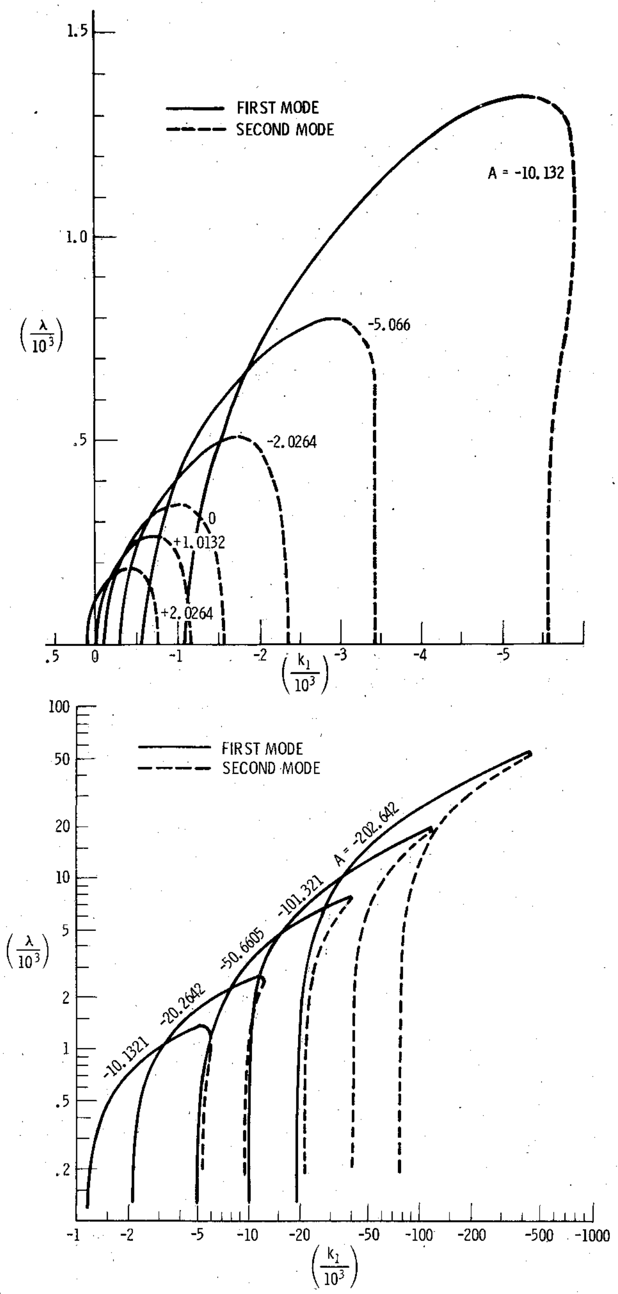

Fig. 2 Real eigenvalues.

where $p_{1,2} = \sigma \pm \beta$, $p_{3,4} = \sigma \pm i\delta$.

$$\beta = \left(\frac{\lambda}{4\sigma} - \sigma^2 - \frac{A\pi^2}{2}\right)^{1/2}, \qquad \delta = \left(\frac{\lambda}{4\sigma} + \sigma^2 + \frac{A\pi^2}{2}\right)^{1/2}$$
$$k = k_1 + ik_2 = 4\left(\sigma^2 - \frac{A\pi^2}{4}\right)^2 - \left(\frac{\lambda}{4\sigma}\right)^2 \tag{16}$$

Flutter Analysis‡

By selecting values of A and λ, then varying σ until Eq. (15) is satisfied, the eigenvalues k_1 and k_2 are obtained. Figures 2a, 2b, 3, and 4 show the trends. Once the eigenvalues are extracted, it is then possible to present the flutter boundaries. First, the conditions on the total damping for flutter are established by solving Eq. (12b) for Ω in terms of the other generic variables. Then, by solving the quadratic equation,

‡Since the approach in this analysis was also used by Dugundji[11] for isotropic panels, materials that might unnecessarily repeat this reference are omitted.

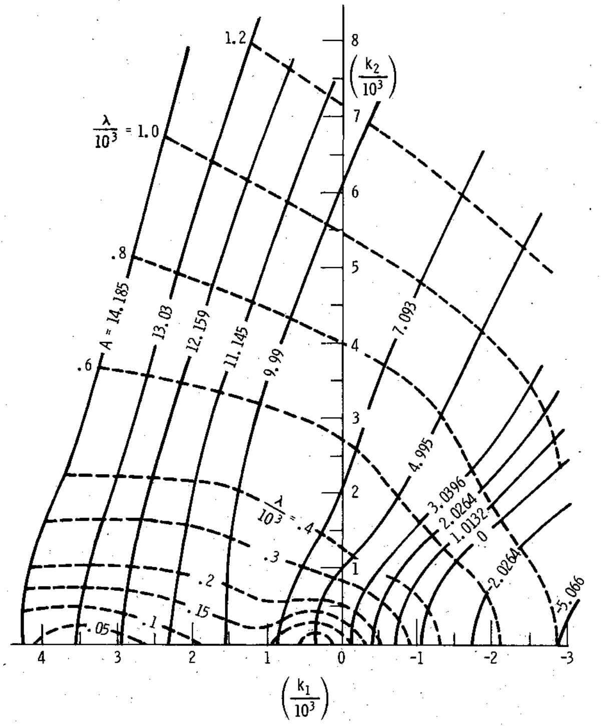

Fig. 3 Complex eigenvalues.

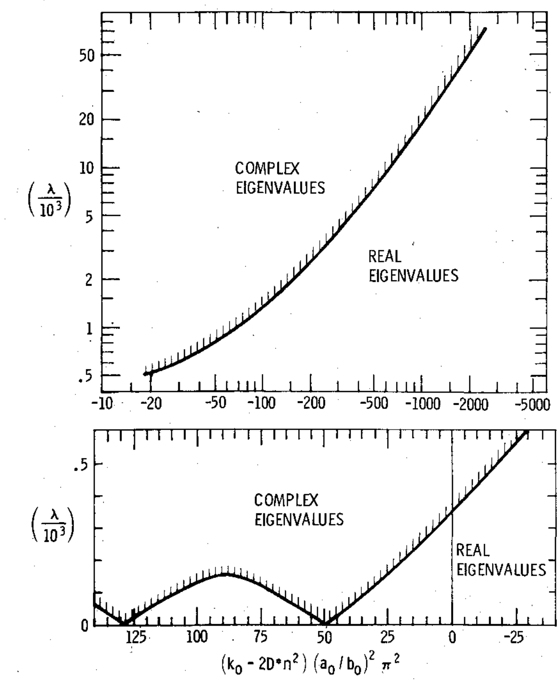

Fig. 4 Minimum value of $\lambda/10^3$ for the first appearance of complex eigenvalues (flutter for $g_T = 0$).

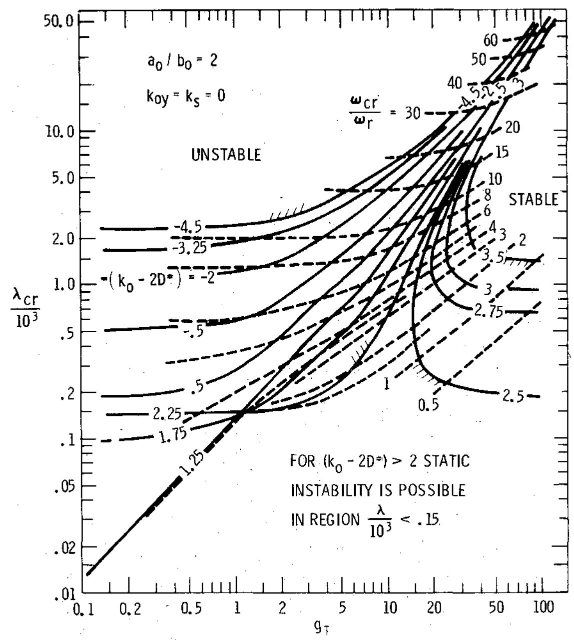

Fig. 5 Effects of damping and midplane forces on the flutter boundaries.

the real and imaginary parts of Ω are given by

$$\alpha = \tfrac{1}{2}\left[-g_T \pm \sqrt{2}\left\{+\left(\left[(g_T/2)^2 + f_1\right]^2 + f_2^2\right)^{1/2} + (g_T/2)^2 + f_1\right\}^{1/2}\right] \quad (17a)$$

$$\frac{\omega}{\omega_r} = \pm \frac{\sqrt{2}}{4} f_2 \left\{+\left(\left[(g_T/2)^2 + f_1\right]^2 + f_2^2\right)^{1/2} + (g_T/2)^2 + f_1\right\}^{1/2} \quad (17b)$$

where

$$f_1 = k_1/\pi^4 - n^4 (a_0/b_0)^4 - k_s + n^2 k_{0y}, \quad f_2 = k_2/\pi^4 \quad (18)$$

Since flutter oscillations are characterized by $\alpha \geq 0$, routine algebraic manipulation of Eq. (17a) results in

$$g_T^2 \geq f_2^2 / -f_1 \quad (19)$$

Equation (19), which gives the Houbolt parabola (first introduced in Ref. 9) defines the condition on the total damping coefficient for flutter. The corresponding flutter frequency is given by

$$\omega_{cr}/\omega_r = (-f_1)^{1/2} \quad (20)$$

By keeping certain generic variables constant and varying the others, Eq. (19) can be satisfied. Hence, plots illustrating the effects of the various generic variables on the flutter boundaries can be shown (see Figs. 5-10). Although it is possible to use na_0/b_0 as an effective aspect ratio, the analysis uses $n = 1$ in generating the plots. For $g_T = 0$, flutter occurs when two undamped natural frequencies coalesce; for $g_T > 0$, flutter sets in at a higher dynamic pressure.

Effects of Generic Variables on Flutter Boundaries§

To see the effects of individual generic variables on the flutter boundaries clearly, it is necessary to keep the others constant. The results are as follows.

§The generic variables used in computing the critical quantities in this paper are chosen to show that the plots for isotropic panels in Ref. 11 can be generalized and used for *all* orthotropic panels.

Figures 5 and 6 depict the effects of damping and midplane forces. Static and dynamic instabilities are possible. The static instability sets in when $(k_0 - 2D^*) \geq 2$. The value 2 is also the minimum of $(k_0 - 2D^*)$ for static instability for $a_0/b_0 = m$, where $m \geq 1$ (Refs. 4 and 6). For $(k_0 - 2D^*) < 2.4$ an increment in g_T results in a higher λ_{cr}, whereas for $(k_0 - 2D^*) > 2.4$ an increment in g_T can result in a lower λ_{cr}. Thus, it is seen from these figures that the type of damping used in this analysis could be stabilizing or destabilizing, depending on the values of $(k_0 - 2D^*)$, g_T, and λ_{cr}.

The effects of damping and aspect ratio are shown in Fig. 7. Since $(k_0 - 2D^*) < 2$, no static instability is possible. As can be seen in Fig. 8, which shows the effects of damping and spring foundation, $(k_0 - 2D^*) < 2$; hence only dynamic instability is

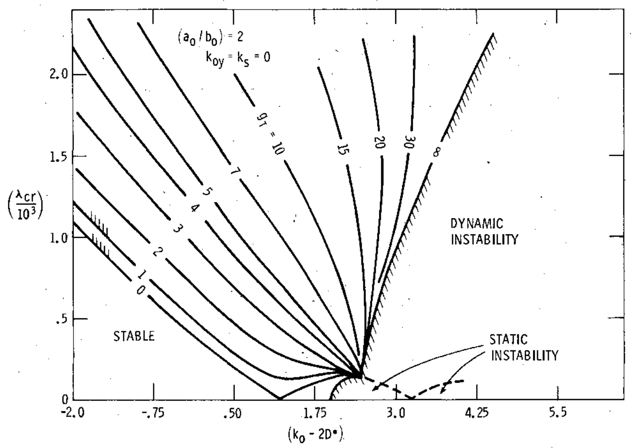

Fig. 6 Effects of damping and midplane forces on the flutter boundaries.

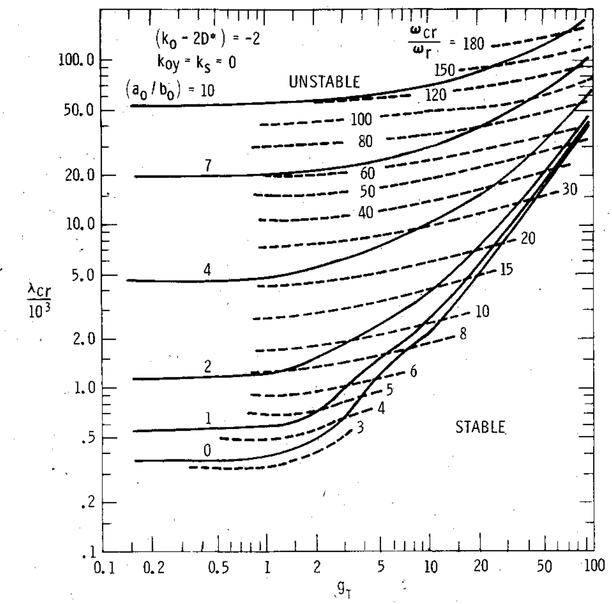

Fig. 7 Effects of damping and aspect ratio on the flutter boundaries.

possible. Figure 9 shows the deflection mode shapes for various generic variables.

Traveling Wave Theory

A panel with low aspect ratio (b_0/a_0) is considered here as an infinite strip with finite b_0.[10,11] For this analysis Eqs. (5) are modified as follows

$$x_0 = b_0\xi_0, \quad y = b_0\eta, \quad \tilde{\omega}_r \triangleq [\pi^4/\rho h b_0^4]^{1/2}, \quad \tau = \tilde{\omega}_r t \tag{21}$$

Substituting Eqs. (21) into Eqs. (3) results in

$$\frac{\partial^4 w}{\partial\xi_0^4} + 2D^* \frac{\partial^4 w}{\partial\xi_0^2\partial\eta^2} + \frac{\partial^4 w}{\partial\eta^4} + \pi^2 k_0 \frac{\partial^2 w}{\partial\xi_0^2} + \pi^2\left(\frac{b_0}{a_0}\right)^2 k_{0y}\frac{\partial^2 w}{\partial\eta^2}$$
$$+ \tilde{\lambda}\frac{\partial w}{\partial\xi} + \frac{\pi^4\partial^2 w}{\partial\tau^2} + \pi^4\tilde{g}_T\frac{\partial w}{\partial\tau} + \pi^4\tilde{k}_s w = 0 \tag{22}$$

where

$$\tilde{k}_s = \left(\frac{b_0}{a_0}\right)^4 k_s, \quad \tilde{g}_T = \left(\frac{b_0}{a_0}\right)^2 g_T, \quad \tilde{\lambda} = \left(\frac{b_0}{a_0}\right)^3 \lambda \tag{23}$$

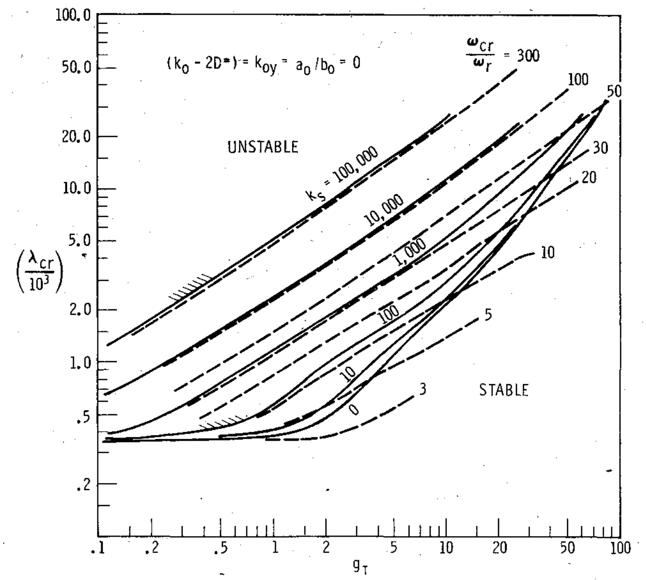

Fig. 8 Effects of damping and spring foundation on the flutter boundaries.

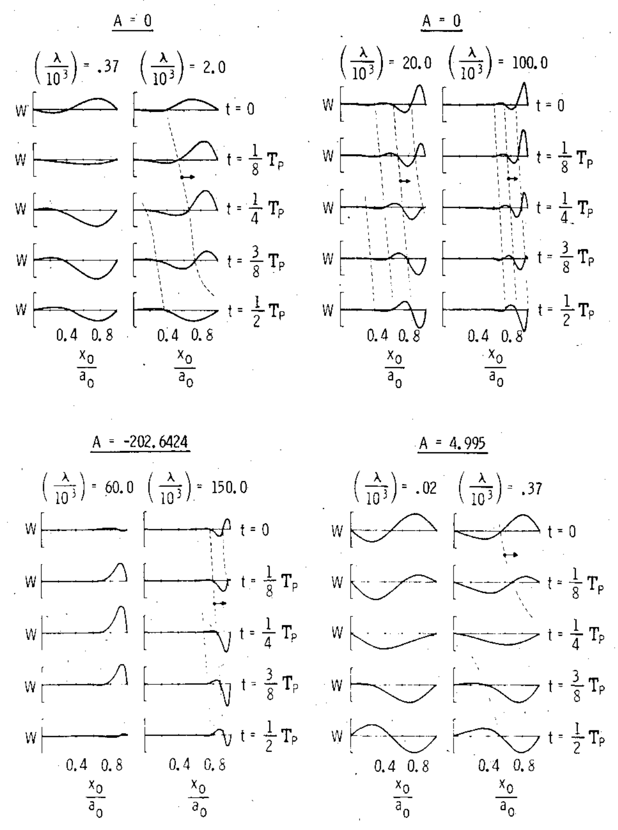

Fig. 9 Modes shapes.

By seeking traveling wave solutions of the form

$$w = W_0(\sin n\pi\eta)\exp\left[\frac{2\pi i}{\ell_0}\left(\frac{c}{\tilde{\omega}_r}\tau - b_0\xi_0\right)\right]$$

to Eq. (22) and the corresponding boundary conditions, the condition for a nontrivial solution becomes

$$\left(\frac{c}{c_r}\right)^2 - i(\tilde{g}_T\ell_0/b_0)\left(\frac{c}{c_r}\right) - T^2 + i(\tilde{\lambda}\ell_0/8\pi^3 b_0) = 0 \tag{24}$$

where

$$T^2 = \tfrac{1}{4}\,[n^4(\ell_0/2b_0)^2 - (k_0 - 2n^2D^*) + (2b_0/\ell_0)^2 + (\ell_0/2b_0)^2\tilde{k}_s - n^2(b_0/a_0)^2(\ell_0/2b_0)^2 k_{0y}], \quad c_r = 2b_0\tilde{\omega}_r/\pi \tag{25}$$

ℓ_0 and c are the wavelength and complex wave speed, respectively. The reference wave speed c_r can be physically interpreted as the minimum vacuum wave speed possible for a panel with $k_s = k_{0y} = 0$, $(k_0 - 2n^2D^*) = -2n^2$. Solving Eq. (24) for c results in

$$c/c_r = c_1 + ic_2 \tag{26}$$

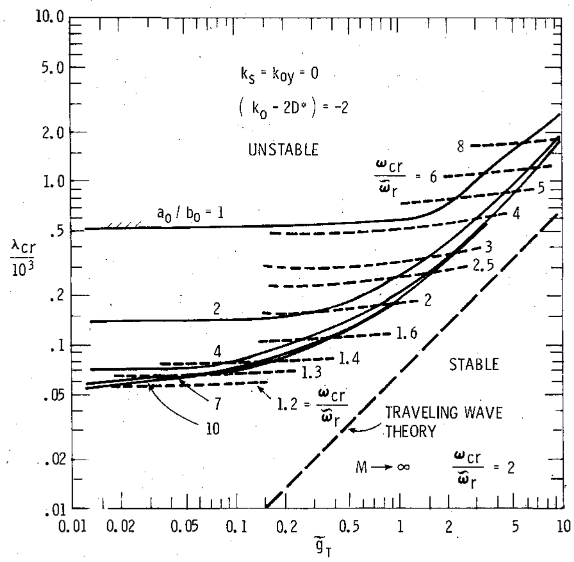

Fig. 10 Finite panel vs traveling wave theories.

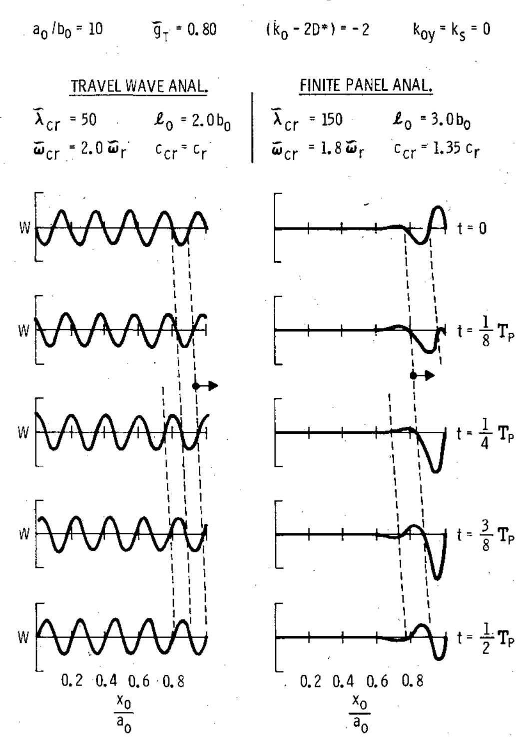

Fig. 11 Finite panel vs traveling wave theories.

where

$$c_2 = (\tilde{g}_T\ell_0/8b_0) \pm \frac{\sqrt{2}}{2}\left(+\left[\{T^2 - (\tilde{g}_T\ell_0/8b_0)^2\}^2 + (\tilde{\lambda}\ell_0/8\pi^3 b_0)^2\right]^{1/2} - \left[T^2 - (\tilde{g}_T\ell_0/8b_0)^2\right]\right)^{1/2}$$

$$c_1 = -[\tilde{\lambda}\ell_0/16\pi^3 b_0]/[c_2 - (\tilde{g}_T\ell_0/8b_0)] \tag{27}$$

By plotting variations of c with $\tilde{\lambda}$ for various wavelengths $\ell_0/2b_0$, the complete panel behavior can be seen. Instability is assumed to occur when $c_r c_2 \le 0$. Hence for flutter [using Eqs. (26) and (27)],

$$\tilde{\lambda}_{cr} = 2\pi^3 T\tilde{g}_T \tag{28}$$

or

$$U_{cr} = (M^2 - 2)/(M^2 - 1)c_r T \tag{29}$$

At flutter,

$$c_{cr} = +Tc_r, \quad \omega_{cr} = 2\pi Tc_r/\ell_0 = 2(2b_0/\ell_0)T\tilde{\omega}_r \tag{30}$$

For $k_s = k_{0y} = 0$, $(k_0 - 2n^2D^*) = -2n^2$, the minimum value of T, 1, occurs for $n = 1$, $\ell_0/2b_0 = 1$. Hence from Eqs. (28-30) the generic variables for the first onset of flutter are realized. Figures 10 and 11 compare the finite panel and traveling wave theories.

Concluding Remarks

This paper has attempted to present a unified panel flutter theory at high supersonic Mach numbers including viscous damping effects for orthotropic and isotropic panels. The theory which evolves from the orthotropic physical space relies on generic variables derived via affine tranformations. Classical plate and two-dimensional first-order aerodynamic theories are used. A closed-form solution of the resulting partial differential equations reveals the nature of the eigenvalues and their general behavior. Consequently, generic flutter boundaries, having the results of Ref. 11 (for isotropic panels) as a subset, are presented. Viscous damping, which seems to be important for understanding the unified panel flutter theory, can be destabilizing. The standing and traveling wave flutter theories are also compared.

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